C2 Factor and Remainder Theorem

1. June 2010 qu.1

The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + ax^2 - ax - 14$, where a is a constant.

(i) Given that $(x - 2)$ is a factor of $f(x)$, find the value of a. [3]

(ii) Using this value of a, find the remainder when $f(x)$ is divided by $(x + 1)$. [2]

2. Jan 2010 qu.6

The cubic polynomial $f(x)$ is given by $f(x) = 2x^3 + ax^2 + bx + 15$,

where a and b are constants. It is given that $(x + 3)$ is a factor of $f(x)$ and that, when $f(x)$ is divided by $(x - 2)$, the remainder is 35.

(i) Find the values of a and b. [6]

(ii) Using these values of a and b, divide $f(x)$ by $(x + 3)$. [3]

3. June 2009 qu.7

The polynomial $f(x)$ is given by $f(x) = 2x^3 + 9x^2 + 11x - 8$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. [2]

(ii) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$. [2]

(iii) Express $f(x)$ as a product of a linear factor and a quadratic factor. [3]

(iv) State the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. [2]

4. Jan 2009 qu.9

(i) The polynomial $f(x)$ is defined by $f(x) = x^3 - x^2 - 3x + 3$.

Show that $x = 1$ is a root of the equation $f(x) = 0$, and hence find the other two roots. [6]

(ii) Hence solve the equation $\tan^3 x - \tan^2 x - 3\tan x + 3 = 0$

for $0 \le x \le 2\pi$. Give each solution for x in an exact form. [6]

5. June 2008 qu.4

The cubic polynomial $ax^3 - 4x^2 - 7ax + 12$ is denoted by $f(x)$.

(i) Given that $(x - 3)$ is a factor of $f(x)$, find the value of the constant a. [3]

(ii) Using this value of a, find the remainder when $f(x)$ is divided by $(x + 2)$. [2]

6. June 2007 qu.9

The polynomial f(x) is given by $f(x) = x^3 + 6x^2 + x - 4$.

(i) (a) Show that $(x + 1)$ is a factor of f(x). [1]

(b) Hence find the exact roots of the equation f(x) = 0. [6]

(ii) (a) Show that the equation $2\log_2(x + 3) + \log_2 x - \log_2 (4x + 2) = 1$

can be written in the form f (x) = 0. [5]

(b) Explain why the equation $2\log_2(x + 3) + \log_2 x - \log_2 (4x + 2) = 1$

has only one real root and state the exact value of this root. [2]

7. Jan 2007 qu.8

The polynomial f(x) is defined by $f(x) = x^3 - 9x^2 + 7x + 33$.

(i) Find the remainder when f(x) is divided by $(x + 2)$. [2]

(ii) Show that $(x - 3)$ is a factor of f(x). [1]

(iii) Solve the equation f(x) = 0, giving each root in an exact form as simply as possible. [6]

8. June 2006 qu.8

The cubic polynomial $2x^3 + ax^2 + bx - 10$ is denoted by f(x). It is given that, when f(x) is divided by $(x - 2)$, the remainder is 12. It is also given that $(x + 1)$ is a factor of f(x).

(i) Find the values of a and b. [6]

(ii) Divide f(x) by $(x + 2)$ to find the quotient and the remainder. [5]

9. Jan 2006 qu.8

The cubic polynomial $2x^3 + kx^2 - x + 6$ is denoted by f(x). It is given that $(x + 1)$ is a factor of f(x).

(i) Show that $k = -5$, and factorise f(x) completely. [6]

(ii) Find $\int_{-1}^{2} f(x)\, dx$. [4]

(iii) Explain with the aid of a sketch why the answer to part (ii) does not give the area of the region between the curve $y = f(x)$ and the x-axis for $-1 \le x \le 2$. [2]

10. June 2005 qu.5

The cubic polynomial f(x) is given by $f(x) = x^3 + ax + b$,

where a and b are constants. It is given that $(x + 1)$ is a factor of f(x) and that the remainder when f (x) is divided by $(x - 3)$ is 16.

(i) Find the values of a and b. [5]

(ii) Hence verify that f (2) = 0, and factorise f (x) completely. [3]